

Measuring the Degree of Time Varying Market Inefficiency *

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Abstract

We estimate a time varying autocorrelation of stock returns as a degree of market inefficiency; the relative inefficiency of the U.S. stock market varies from 1955 to 2006.

Keywords: Efficient market hypothesis; Degree of market inefficiency; State space model; Kalman smoothing

JEL classification: C22; G14

1 Introduction

Although the Efficient Market Hypothesis (EMH) has been a source of a number of articles since Samuelson (1965) introduced the concept of fair game to financial economics, the EMH is still in controversy. Researchers' views on the EMH have changed during the past 40 years.

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Fama (1970) summarized that, in early researches, many articles supported the EMH; Fama (1991) himself modified his view because of a lot of anomalies reported after Fama (1970). Even in recent years, Malkiel (2004)[ch.11] sticks to his stance that the EMH is almost true; Shiller (2005)[ch.10] is skeptical of the hypothesis. Today nobody can summarize the dispute over the EMH in a couple of lines. The battle between proponents of the EMH and advocates of behavioral finance is still ongoing and, the authors believe, will never end.

We assert that any effort to verify the EMH by the classical hypothesis testing is in vain and that it is productive to establish a measure of relative market inefficiency as Campbell, Lo and MacKinlay (1997)[1.5.2] point out. The readers can find recent literature which compares relative performance of market efficiency, but there is little literature that studies the relative market efficiency through time based on time series data. Lo (2004) is an exception, as is described in section 2. In this paper, we realize an empirical study measuring a gradually time varying structure of market inefficiency.

We deal with the EMH in the weak sense in financial markets; it implies one can never predict returns of an asset by analyzing past data. Inefficiency can be interpreted as implying existence of exploitable opportunities. While some degree of serial correlation implies the predictability, it may not imply inefficiency if the predictability is insufficient to overcome transaction costs. We also note that delayed trading of some stocks after a shock may impart specious autocorrelation to an index. However, in case of monthly observations, we consider an autocorrelation of stock returns as a good proxy of market inefficiency when we deal with the EMH in the weak sense.

Our task is to measure the autocorrelation of the stock returns *in each period* as a degree of market inefficiency, supposed to be time varying. We adopt a time varying AR model, in which the AR coefficients can vary over time. Applying a state space model, we estimate the AR coefficients by the Kalman smoothing.

2 Empirical Method

Our method consists of two steps: (1) checking a time varying structure of autocorrelations of stock returns data based on the Moving Window method and (2) estimating the time varying AR(1) coefficients by using a state space model. This paper uses the monthly

returns for the S&P500 stock index from Jan 1955 to Feb 2006, taken from Robert Shiller’s website. In practice, we take log first difference of timeseries of the stock index.

We preliminarily apply the Moving Window method to the data and calculate the first order autocorrelations, as Lo (2004) do. Let (x_1, \dots, x_T) be a sequence of the stock returns. When using the Moving Window method, one makes $T-w+1$ sub samples $(x_{\tau-w+1}, \dots, x_\tau)$ for $\tau = w, \dots, T$ with sub sample size w , called a window width, and then derives the first order autocorrelation to each sub sample.

Figure 1 exhibits the result, where the window width is 100. The autocorrelations of stock returns present a time varying structure of the market. This result itself implies that a degree of market inefficiency varies through time, as Lo argues. We measure it in more sophisticate way than he does.

Since the time series shown in Figure 1 may contain a unit root, we apply the Augmented Dickey Fuller (ADF) test to check the existence of a unit root. We assume a model with time trend and a constant and use Schwarz Bayesian Information Criterion (SBIC) as an order selection criterion. Lag 0 is chosen and the test statistics is computed to be -2.70 . The corresponding 5% critical value is -3.42 . Thus we can not reject the null that the data contain a unit root. This fact supports our assumption that the AR coefficients in a time varying AR model follow a random walk process.

(Figure 1 around here.)

The simplest method to measure the autocorrelations of stock returns is to apply the AR model to the stock returns data, since the Yule-Walker equation assures sample autocorrelation functions correspond to coefficients of the AR model with each other uniquely.

We here show a state space model representation of the time varying AR model, which allows AR coefficients to vary through time. The time varying AR model is a specific case of the state space representation of a dynamical system that is composed of an *observation equation* and a *transition equation* (See Hamilton (1994)[Ch13] for state space models). One should formulate the transition equation, representing dynamics of AR coefficients, based on *a priori* information. Here, our idea is to formulate the transition equation by studying the moving autocorrelation series calculated by the Moving Window method above. We have already seen that time series of the autocorrelations calculated by the Moving Window method contain a unit root. Thus,

we assume the time varying AR coefficients contain a unit root. This assumption makes our state space model so simple as shown in (1) and (2).

observation equation

$$x_t = \begin{pmatrix} x_{t-1} & x_{t-2} & \cdots & x_{t-k} \end{pmatrix} \begin{pmatrix} \alpha_{1,t} \\ \alpha_{2,t} \\ \vdots \\ \alpha_{k,t} \end{pmatrix} + u_t, \quad u_t \sim \mathcal{N}(0, \sigma_{u_t}^2) \quad (1)$$

transition equation

$$\begin{pmatrix} \alpha_{1,t} \\ \alpha_{2,t} \\ \vdots \\ \alpha_{k,t} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} \begin{pmatrix} \alpha_{1,t-1} \\ \alpha_{2,t-1} \\ \vdots \\ \alpha_{k,t-1} \end{pmatrix} + \begin{pmatrix} v_{1,t} \\ v_{2,t} \\ \vdots \\ v_{k,t} \end{pmatrix}, \quad \mathbf{v}_t \sim \mathcal{N}_k(\mathbf{0}, \sigma_{v_t}^2 \mathbf{I}) \quad (2)$$

Note $\mathbf{v}_t \equiv (v_{1,t} \ v_{2,t} \ \cdots \ v_{k,t})'$ in (2).

In practice, we use a new calculation technique to realize Kalman smoothing developed by Ito (2007). Note that Ito's new calculation technique leads to exactly the same estimation result as the traditional Kalman smoothing. It merely cuts down calculation time by doing vast amounts of calculation at a time.

3 Empirical Result

Applying the model described in the previous section to the monthly returns for the S&P500 stock index, we derive the relative degree of market inefficiency in each period, which is exhibited in Figure 2. The figure represents the time varying AR coefficients. We let the initial values both for state variables and for the returns data to be all 0. Note that the state variable being 0 means that the market is completely efficient. We have confirmed that the result of estimation is robust even when we change initial values of the state variable. Estimated state variables corresponding to the data in earlier periods can be interpreted as a *path for adjustment*. We present here the case of AR(1) model for simplicity.

Figure 2 shows that the estimated AR coefficients vary through time and that the shape of the graph is quite similar to that of autocorrelations estimated by the Moving Window method. Therefore, we insist that the degree of market inefficiency varies through time. Some readers might expect that the stock market is becoming more efficient recently because of development in financial theories as well as in information technologies; we find that the relative degree of market inefficiency varies through time without trend.

In the 1960s and 1970s, the estimated coefficients are relatively at a low level. During the late 1980s, the estimated coefficients climb to the highest level during the whole sample period. Thus we assure that the U.S. stock market is quite inefficient during this period. You can see that the estimated coefficients have falling trend from the late 1990s and go to the lowest level at 2000. Thus, we conclude that the U.S. stock market has become the most efficient at around 2000 in the past half a century.

(Figure 2 around here.)

4 Conclusion

We measure a time varying structure of market inefficiency; we find the degree of market inefficiency varies through time. The U.S. stock market was the most inefficient during the late 1980s and has become the most efficient at around 2000 in the last half a century. When Fama (1970) summarized that many articles supported the EMH, the U.S. stock market was efficient enough. When Fama (1991) summarized that a lot of anomalies were reported after 1970, the U.S. stock market was quite inefficient. A researcher's view on the EMH depends on the level of approximation that one can allow.

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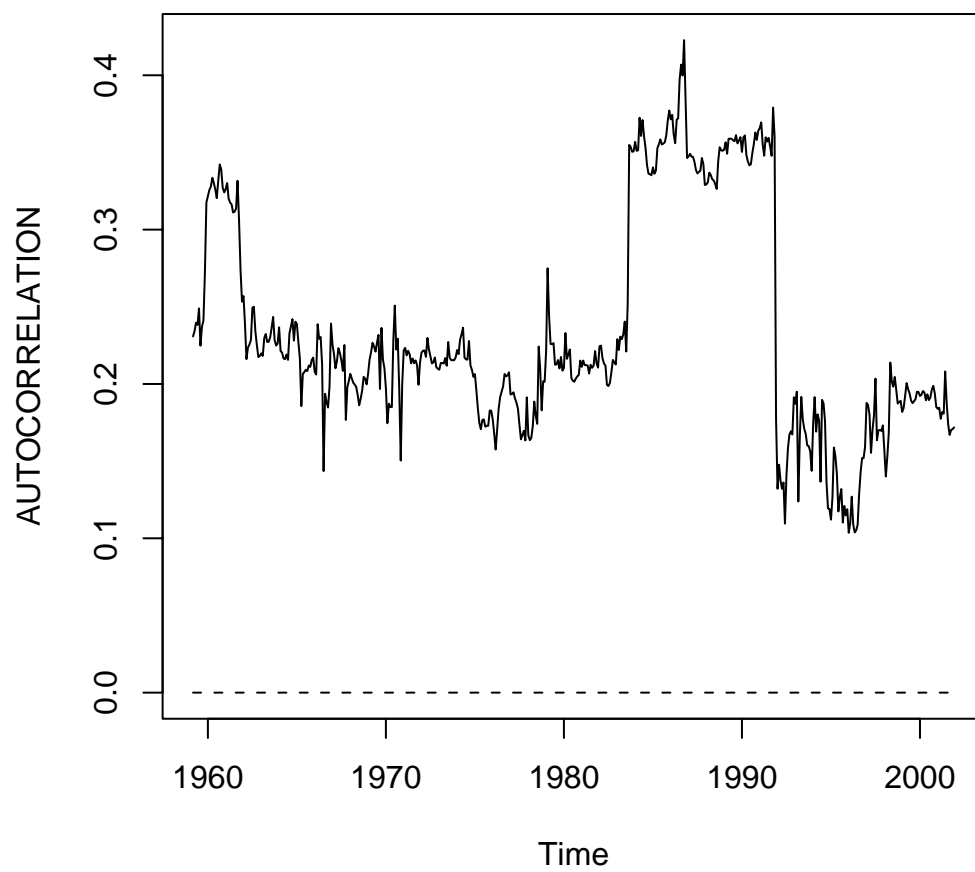


Figure 1: Autocorrelation of Stock Return Series Varies Through Time
We estimate the first order autocorrelations of monthly returns for the S&P500 stock index using the Moving Window method. The window width is 100.

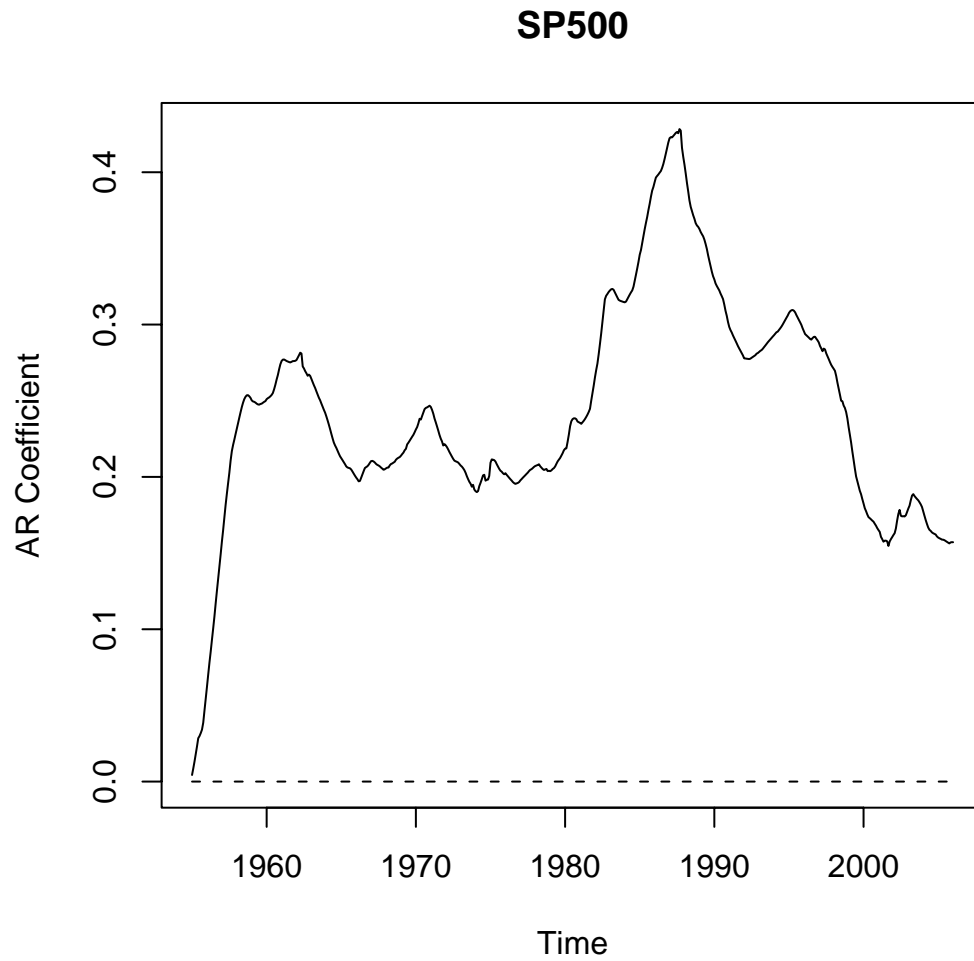


Figure 2: Time Varying Market Inefficiency in the U.S. Stock Market